Conceptul de incertitudine a rezonanţei funcţionale la sistemele dinamice cu acţionare controlată

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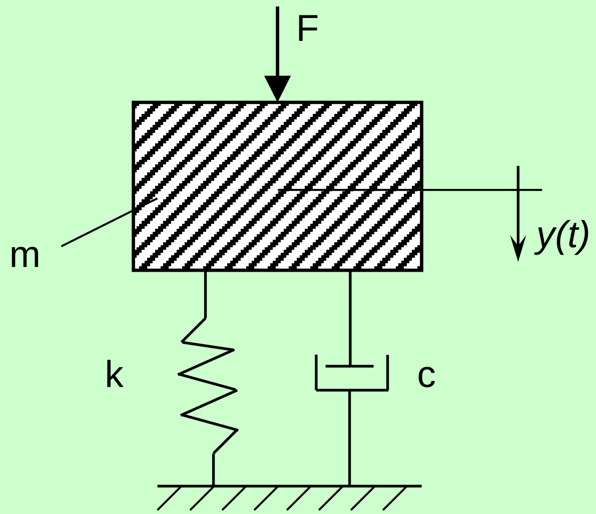
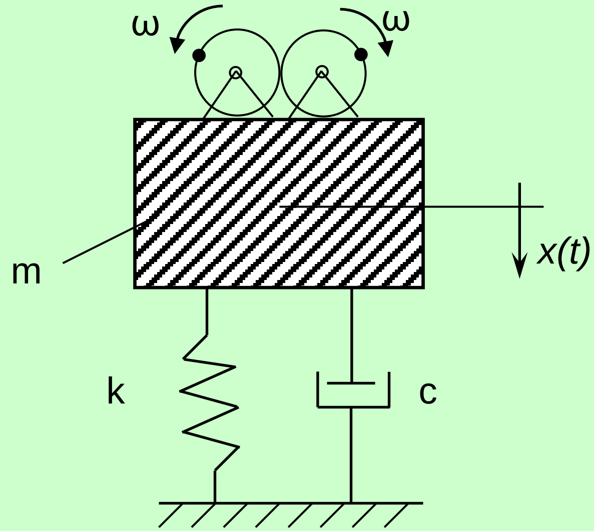
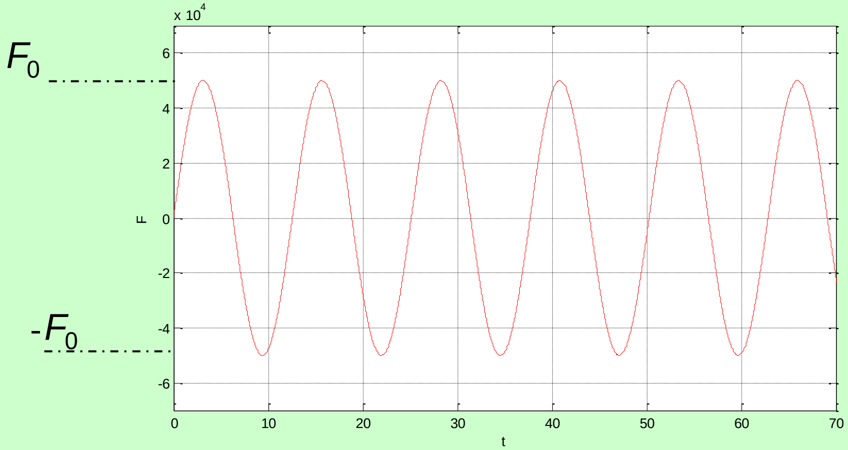
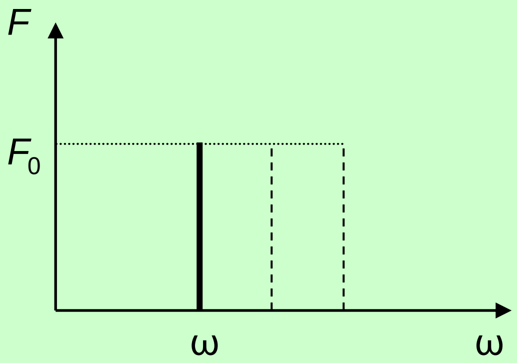
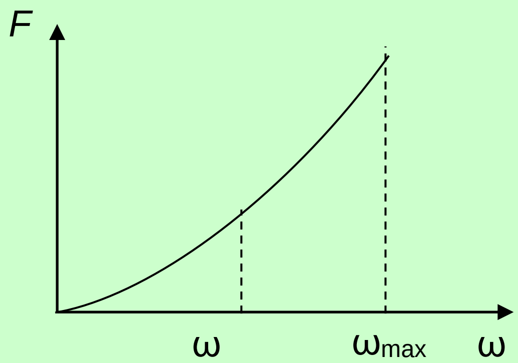
Se prezinta modelul sistemelor dinamice cu functiune in rezonanta a caror instabilitate fizica verifica principiul de incertitudine a lui Hesemberg din mecanica cuantica.

Mentinerea in rezonanta a functionarii sistemelor se poate realiza in mod controlat prin reglaje dar numai cu acceptarea unor criterii de ,,sacrificiu''.

	Specificatie	Sistem dinamic excitat cu forţă perturbatoare	Sistem dinamic excitat cu sistem inerţial rotativ
1	**Modelul dinamic**	 $F(t) = F_0 \sin \omega t\,, \quad F_0 = const$	 $F(t) = m_0 r \omega^2 \sin \omega t$
2	**Sistem liniar vâscoelastic**	m – masa; k – rigiditatea echivalentă; c – vâscozitatea echivalentă	
3	**Excitaţia unidirecţională**	- dinamică, cu forţă aplicată din exterior; $F(t) = F_0 \sin \omega t\,, \qquad F_0 = const$	- dinamică de tip inerţial; $F(t) = 2\dfrac{m_0}{2} r \omega^2 \sin \omega t$ $2\dfrac{m_0}{2} r$ - momentul static al sistemului de dezech.
4	**Degradarea legăturii vâscoelastice**	• Modificarea vâscozităţii echivalente $c(t,\omega) \neq const$. • Modificarea rigidităţii echivalente $k(t,\omega) \neq const$. • Modificarea simultană a vâscozităţii şi rigidităţii $\{c(t,\omega), k(t,\omega)\} \neq const$.	

		Sistem dinamic excitat cu forţă perturbatoare	Sistem dinamic excitat cu sistem inerţial rotativ
5	Răspunsul dinamic staţionar în regim funcţional	$Y = A_1 \sin(\omega t - \varphi_1)$	$X = A_2 \sin(\omega t - \varphi_2)$
		$A_1 = H \dfrac{F_0}{k}$	$A_2 = \dfrac{m_0 r}{m} \Omega^2 H(\Omega, \zeta)$
		$H(\Omega, \zeta) = \dfrac{1}{\sqrt{(1-\Omega^2)^2 + (2\zeta\Omega)^2}}$	$L(\Omega, \zeta) = \Omega^2 H(\Omega, \zeta)$
		$\dfrac{F_0}{k} = \Delta_0$ - deformaţia statică echivalentă	$\dfrac{m_0 r}{m} = A_\infty^{stab}$ - amplitudinea postrezonanţă, $\omega \to \infty$

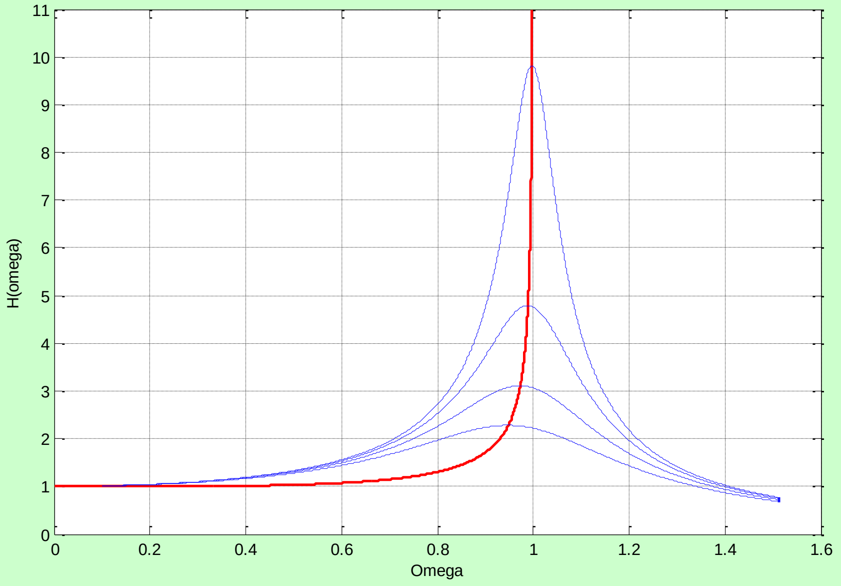
6	Variaţii parametrice în regim forţat	Sistem dinamic excitat cu forţă perturbatoare	Sistem dinamic excitat cu sistem inerţial rotativ
	6.1. Excitaţii	$F(t) = F_0 \sin \omega t$ $F_0 = const$	$F(t) = m_0 r \omega^2 \sin \omega t$ $F_0 = m_0 r \omega^2$
			
			

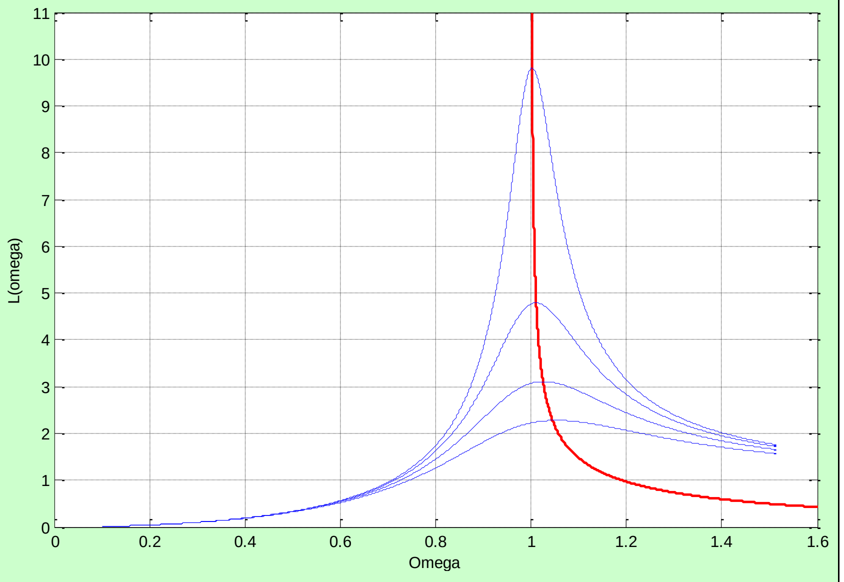
6.2. Funcţii dinamice

$$H(\Omega,\zeta)=\frac{1}{\sqrt{\left(1-\Omega^2\right)^2+\left(2\zeta\Omega\right)^2}}$$

$$L(\Omega,\zeta)=\Omega^2 H(\Omega,\zeta)$$

6.3. Familia de curbe $H=H(\Omega)$ şi $L=L(\Omega)$ cu parametrul $\zeta=0{,}051\ldots0{,}225$





$\Omega=\dfrac{\omega}{\omega_n}$ - pulsaţia relativă de excitaţie;

$\omega_0=\sqrt{\dfrac{k}{m}}$ - pulsaţia proprie în absenţa amortizării;

$\omega_n=\omega_0\sqrt{1-2\zeta^2}$ - pulsaţia proprie naturală în prezenţa amortizării;

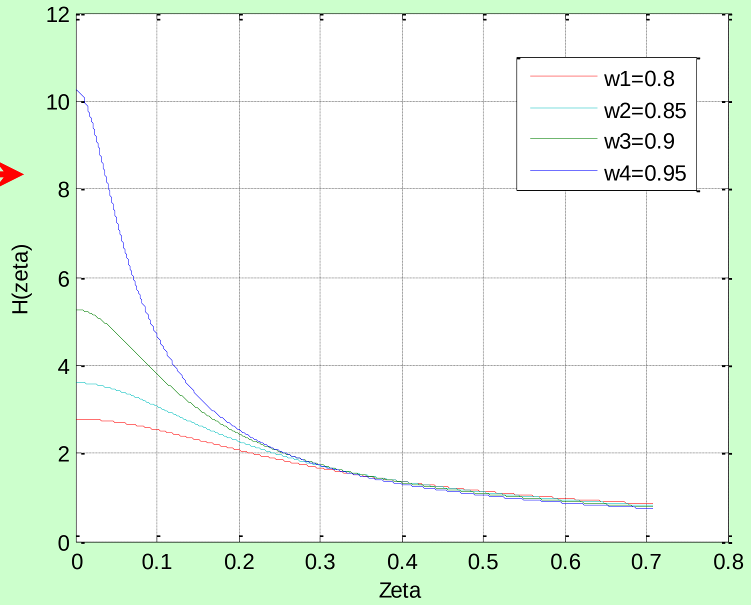
$\zeta=\dfrac{c}{c_{cr}}=\dfrac{c}{2\sqrt{km}}$ - fracţiunea din amortizarea critică pentru sisteme vâscoelastice liniare;

$\Omega_0=\dfrac{\omega}{\omega_0}=1$ - rezonanţa sistemului fără amortizare;

$\Omega_{rez}=\dfrac{\omega}{\omega_n}=1$ - rezonanţa sistemului cu amortizare;

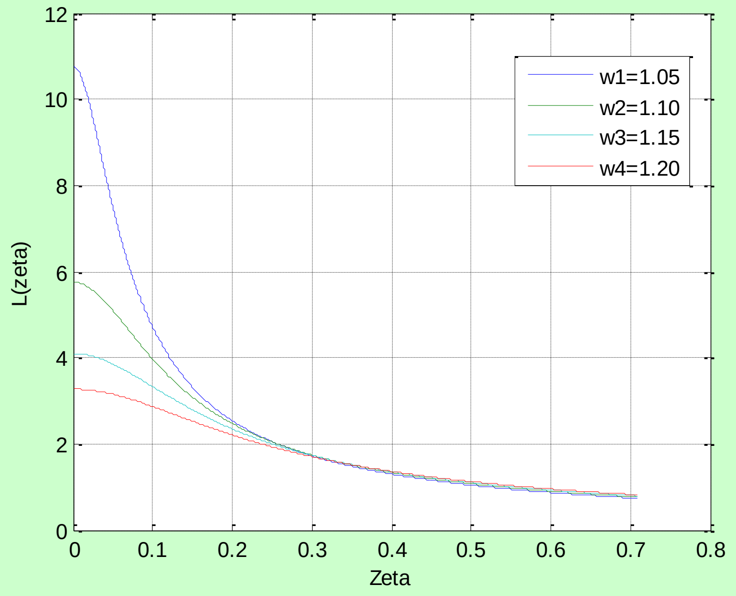
6.4. Familia de curbe $H = H(\zeta)$ cu parametrul $\Omega_1 = 0{,}8 \ldots 0{,}95$

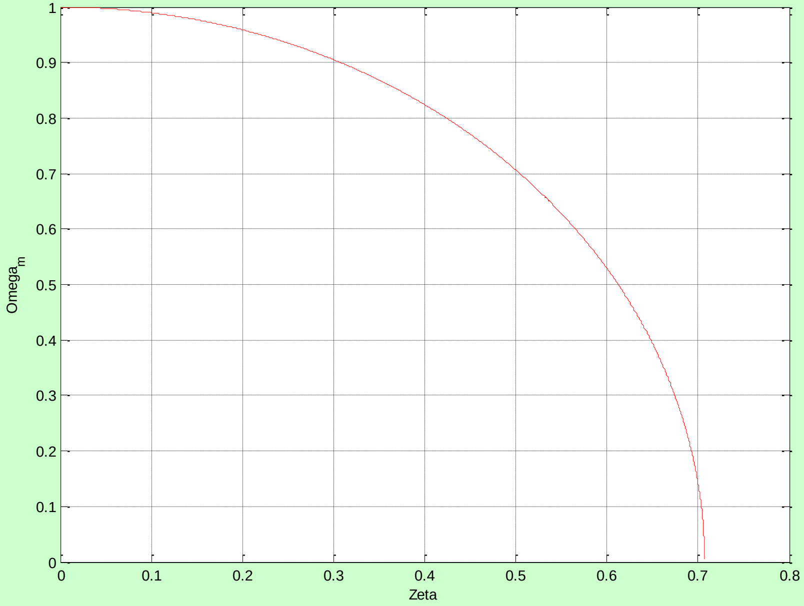
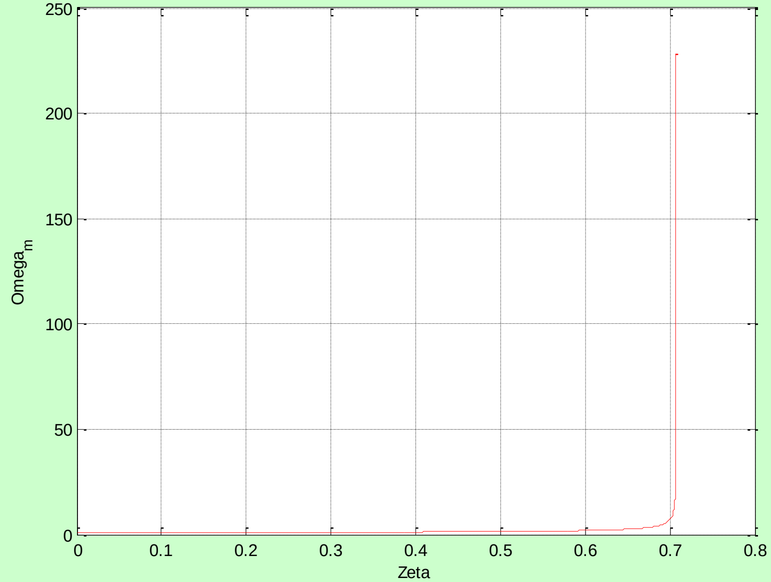
$$H(\Omega, \zeta) = \frac{1}{\sqrt{(1-\Omega^2)^2 + (2\zeta\Omega)^2}}$$



6.5. Familia de curbe $L = L(\zeta)$ cu parametrul $\Omega_2 = 1{,}05;\ 1{,}10;\ 1{,}15;\ 1{,}20$

$$L(\Omega, \zeta) = \Omega^2 H(\Omega, \zeta)$$



7	**Răspunsul în rezonanţă**		
	7.1. Starea de rezonanţă	$\Omega_1 = \Omega = \Omega_{1rez}$ $H(\Omega,\zeta) = H_{\max}^{rez}(\Omega_{1rez})$	$\Omega_2 = \Omega = \Omega_{2rez}$ $L(\Omega,\zeta) = L_{\max}^{rez}(\Omega_{2rez})$
	7.2. Condiţia de concordanţă	$\Omega = \Omega_{1rez} = \sqrt{1-2\zeta^2}$	$\Omega = \Omega_{2rez} = \dfrac{1}{\sqrt{1-2\zeta^2}}$
	7.3. Curba de variaţie		

7.4. Răspunsul maxim la rezonanţă

a) În raport de ζ

- Relaţii analitice

$$H_{max}^{rez} = f(\zeta)$$

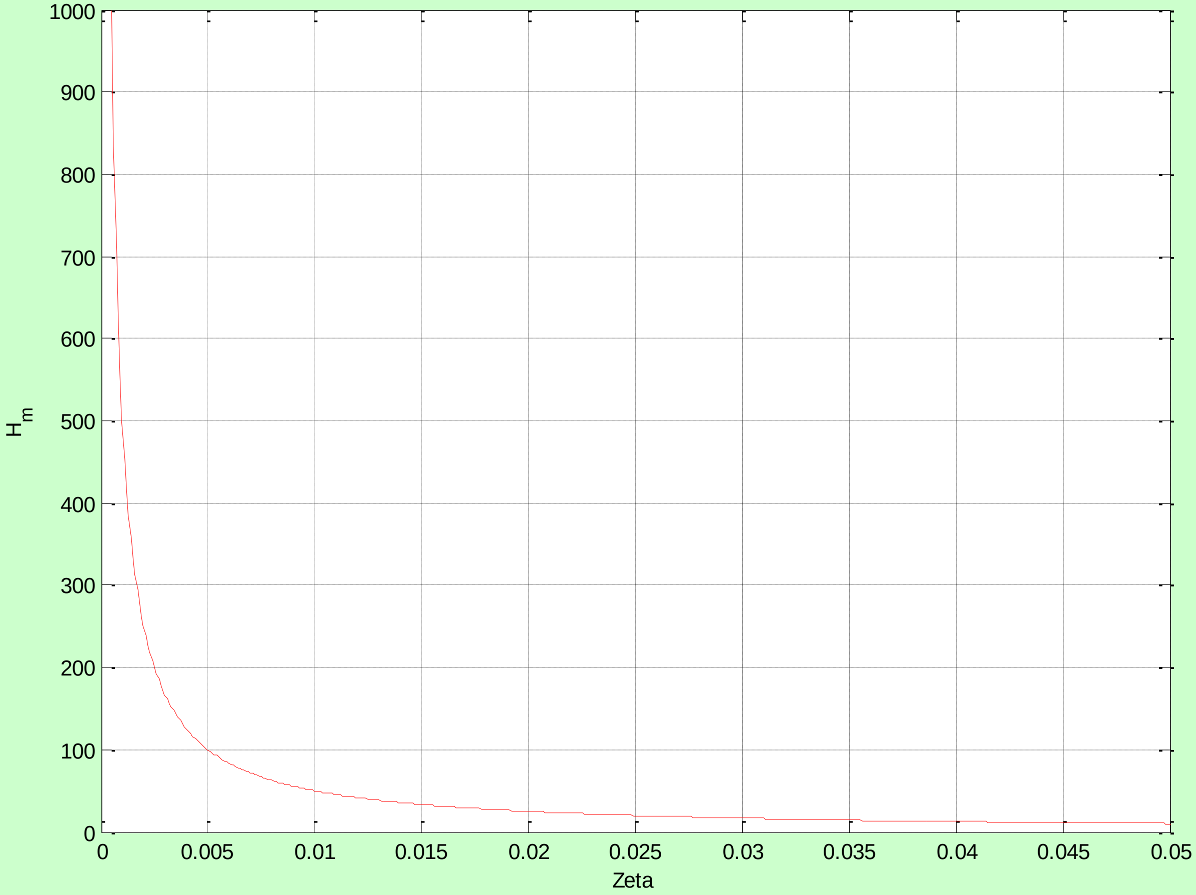
$$H_{max}^{rez} = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

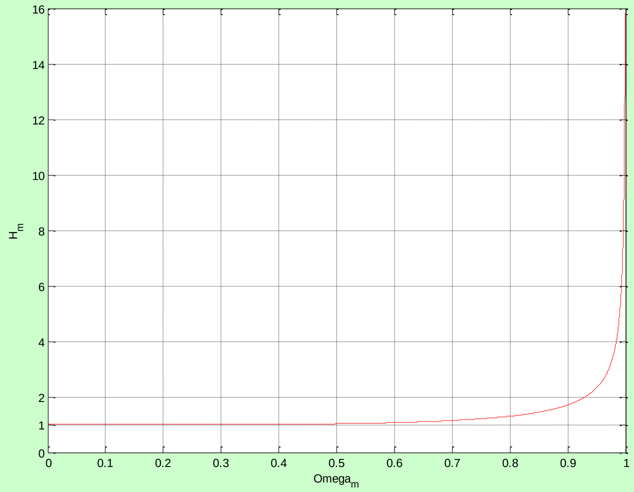
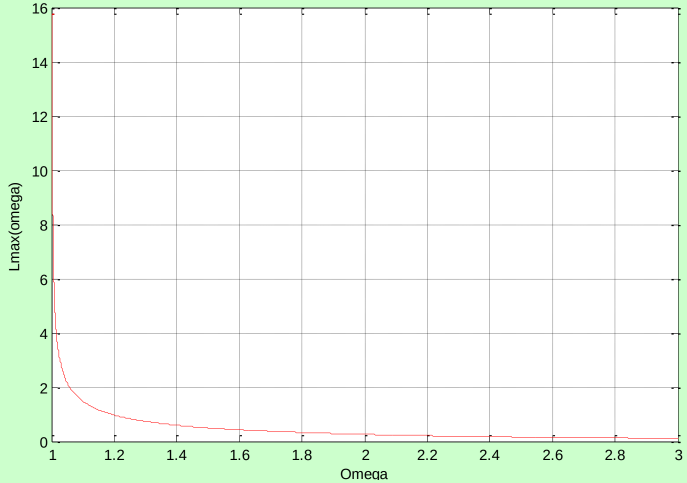
$$L_{max}^{rez} = f(\zeta)$$

$$L_{max}^{rez} = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

- Curbe de variaţie

$$\boxed{H_{max}^{rez} \equiv L_{max}^{rez}}$$

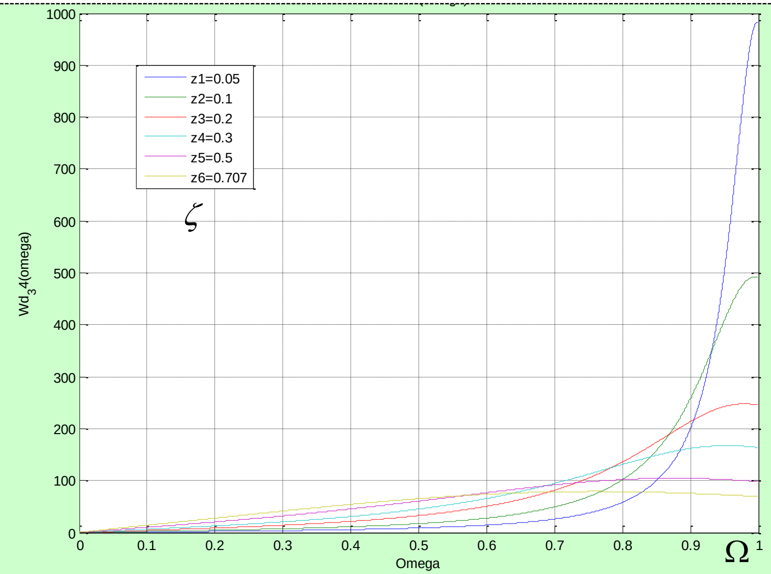


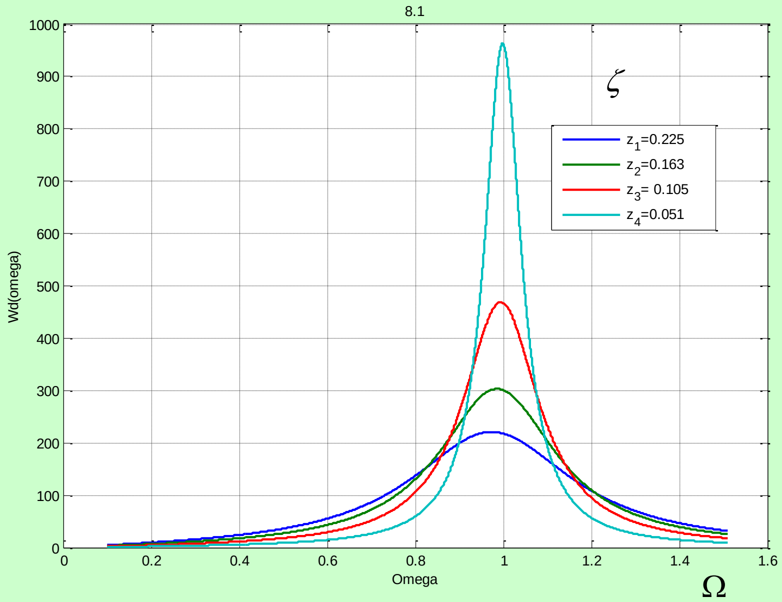
	b) În raport de Ω • Relaţii analitice • Curbe de variaţie	$H_{max}^{rez} = \frac{1}{\sqrt{1-\Omega^4}}$ 	$L_{max}^{rez} = \frac{\Omega^2}{\sqrt{\Omega^4 - 1}}$ 
8	***Energie disipată***	$W_d = 2\pi\left(\frac{F_0}{k}\right)^2 k\zeta\Omega H^2(\zeta,\Omega)$	$W_d = 2\pi\left(\frac{m_0 r}{m}\right)^2 k\zeta\Omega L^2(\zeta,\Omega)$
	8.1. Evaluare în regim dinamic (relaţii analitice)	$\frac{F_0}{k} = \Delta_0 \Rightarrow \boxed{W_d = 2\pi\Delta_0^2 k\zeta\Omega H^2(\zeta,\Omega)}$	$\frac{m_0 r}{m} = A_\infty^{stab} \Rightarrow \boxed{W_d = 2\pi\left(\frac{m_0 r}{m}\right)^2 k\zeta\Omega L^2(\zeta,\Omega)}$

8.2. Familii de curbe

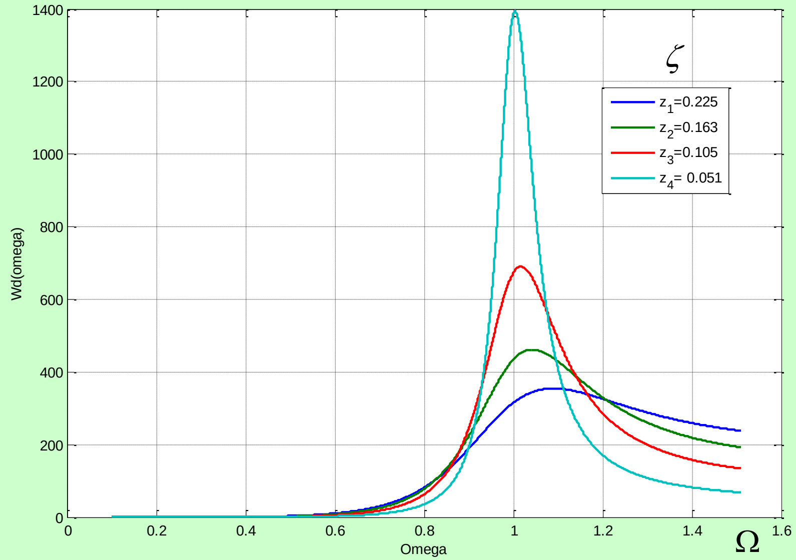
a) Curbe de variaţie în funcţie de Ω, cu parametru ζ.

$$\boxed{W_d = W_d(\Omega)}$$



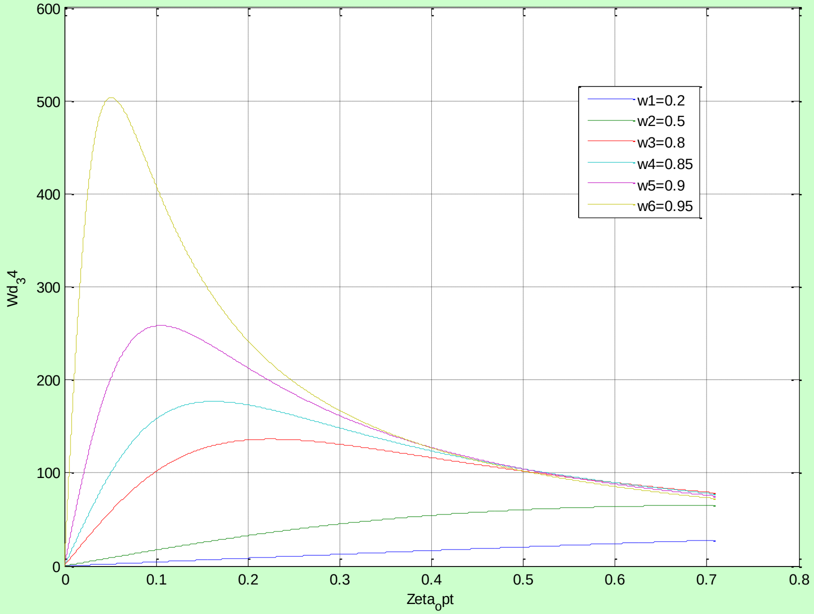


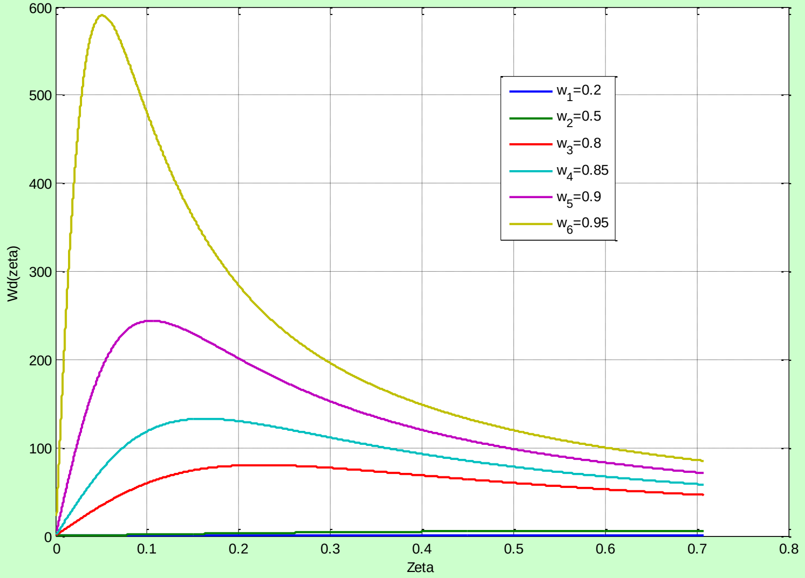
r=0.5 m; m_0=3 Kg;
F_0=50000 N
k=80000000 N/m
m=2000 Kg



b) Curbe de variaţie în funcţie de ζ, cu parametru Ω.

$$W_d = W_d(\zeta)$$

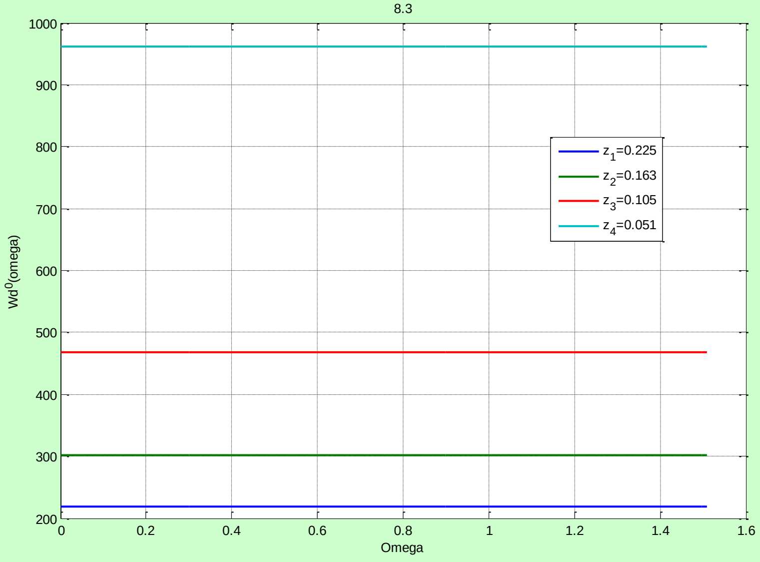




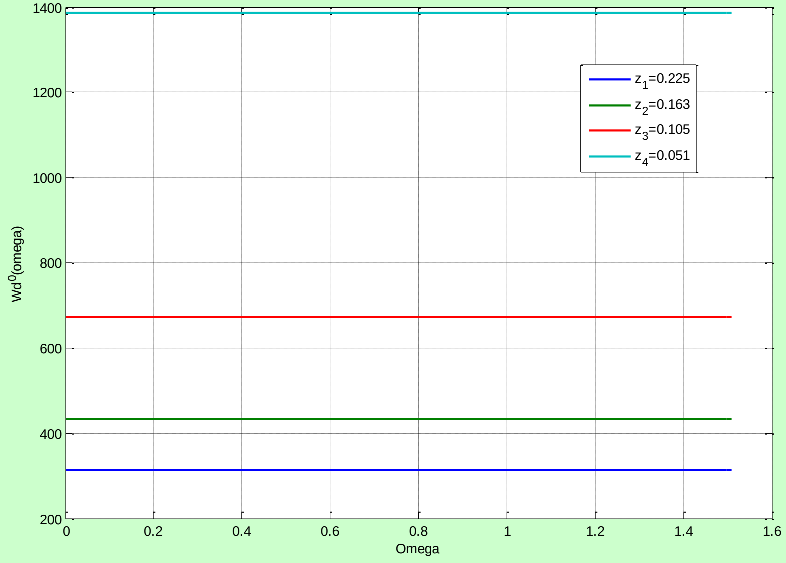
8.3. Evaluarea în regim de rezonanţă

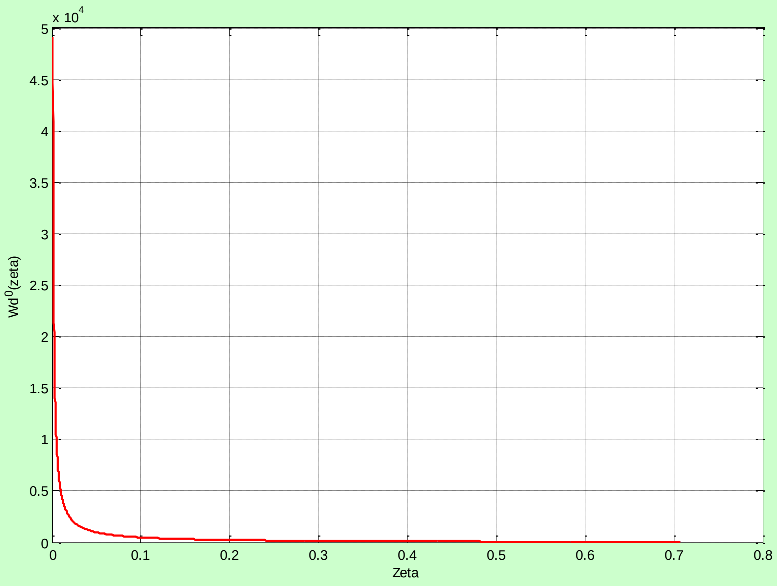
- pentru $\underline{\Omega_0 = 1}$

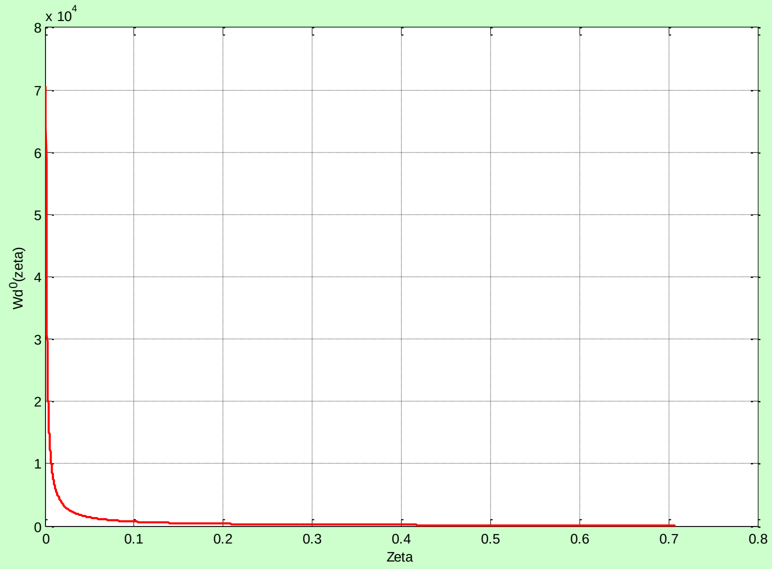
$$W_d^0 = \pi \Delta_0^2 \frac{k}{2\zeta}$$



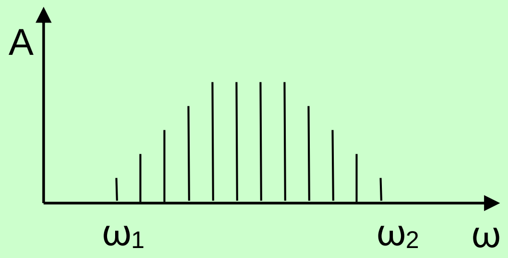
$$W_d^0 = \pi A_\infty^2 \frac{k}{2\zeta}$$







9	**Lărgimea benzii la rezonanţă**	
	9.1.	$\Delta\omega = \frac{1}{\tau} \quad \Rightarrow \boxed{\Delta\omega \cdot \Delta\tau = 1}$ - incertitudinea funcţională la rezonanţă $\tau = \frac{m}{c} = \frac{c}{k} \rightarrow \Delta\tau = \frac{m}{\Delta c} = \frac{\Delta c}{m}$
	9.2.	$\Delta\tau \rightarrow 0 \Rightarrow \Delta c \rightarrow \infty$ variaţia lui c $\Rightarrow \quad \Delta\omega = \frac{\Delta c}{m} \rightarrow \infty$ $\Delta c = nc_1, \; n > 1.$

9.3.	Spectru larg pentru oscilatori multipli $\boxed{W_d(\Delta\omega) >> W_d^{\max}}$		
• pentru Ω_{rez}	$\Omega_1^{rez} = \sqrt{1-2\zeta^2}$ $W_d^{rez} = W_{1d}\left(\zeta, \Omega_1^{rez}\right)$ $W_d^{rez} = 2\pi\Delta_0^2 k\zeta\Omega_1^{rez} H^2\left(\zeta, \Omega_1^{rez}\right)$	$\Omega_2^{rez} = \dfrac{1}{\sqrt{1-2\zeta^2}}$ $W_d^{rez} = W_{2d}\left(\zeta, \Omega_2^{rez}\right)$ $W_d^{rez} = 2\pi A_0^2 k\zeta\Omega_2^{rez} L^2\left(\zeta, \Omega_2^{rez}\right)$	
• pentru $\Omega_{opt}^* \Rightarrow W_d^{\max}$	$\zeta_{1opt} = \dfrac{1-\Omega^2}{2\Omega}$ $\zeta_{1opt} = -\zeta + \sqrt{1+\zeta^2}$ $W_{d1}^{\max} = W_{d1}^{\max}\left(\zeta, \Omega_{1opt}\right)$	$\zeta_{2opt} = \dfrac{\Omega^2-1}{2\Omega}$ $\zeta_{2opt} = +\zeta + \sqrt{\zeta^2+1}$ $W_{d2}^{\max} = W_{d2}^{\max}\left(\zeta, \Omega_{2opt}\right)$	

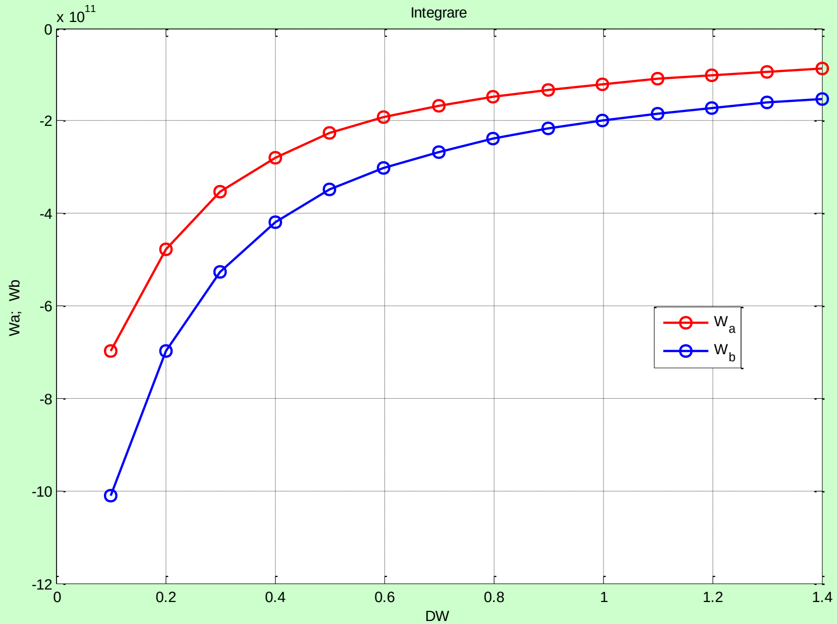
		$$W_d^{total}(\Delta\omega) = \sum_{j=1}^{n} \pi c_n \omega_j A_j^2 = \pi c_n \sum_{j=1}^{n} \omega_j A_j^2$$ $$\boxed{W_d(\Delta\omega) >> W_d(\omega_0)}$$	
	9.4.	$\Delta\tau \to \infty \quad \to \quad \Delta c \to 0$ - fără amortizare $\Delta\omega \to 0 \quad \to \quad \omega = \omega_0$ - oscilator ideal şi unic $$\underline{W_d = mc\,\omega_0^2 A_{rez}^2}$$	
10	**Integrarea**	$$P(\Omega) = \frac{F_0}{k}\omega_0 \frac{\zeta\Omega^2}{(1-\Omega^2)^2 + (2\zeta^2)^2\Omega^2}$$ $$W_a = \int_{\Omega_1}^{\Omega_2} P(\Omega)d\Omega = \int_{1-\Delta\Omega}^{1+\Delta\Omega} P(\Omega)d\Omega$$ $\Delta\Omega = 0.1;\ 0.2;\ 0.3;\ 0.6;\ 1.0;\ 1.4;$	$$P(\Omega) = \frac{(m_0 r)^2 \omega_0^5 \zeta\Omega^4}{k \cdot \left[(1-\Omega^2)^2 + (2\zeta^2)^2\Omega^2\right]}$$ $$W_b = \int_{\Omega_1}^{\Omega_2} P(\Omega)d\Omega = \int_{1-\Delta\Omega}^{1+\Delta\Omega} P(\Omega)d\Omega$$ $\Delta\Omega = 0.1;\ 0.2;\ 0.3;\ 0.6;\ 1.0;\ 1.4;$

$\Delta\Omega = DW$ *; (pe desen)*

F_0=50000 N; k=80000000 N/m;

m=2000 Kg; ω_0 =200 rad/s

r=0.5 m; m_0=3 Kg; $\zeta = 0.3$



ζ =[0.3; 0.225; 0.163; 0.105; 0.051]